The 123 Theorem of Probability Theory and Copositive Matrices

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Abstract

Alon and Yuster give for independent identically distributed real or vector valued random variables X, Y combinatorially proved estimates of the form $\text{Prob}(\|X - Y\| \leq b) \leq c\,\text{Prob}(\|X - Y\| \leq a)$. We derive these using copositive matrices instead. By the same method we also give estimates for the real valued case, involving $X+Y$ and $X-Y$, due to Siegmund-Schultze and von Weizsäcker [3] as generalized by Dong, Li and Li [2]. Furthermore we formulate a version of above inequalities as an integral inequality for monotone functions.

Keywords

Probabilistic inequalities, Copositivity, Integral inequality

References

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